MATH 596 - INTRODUCTION TO MODULAR FORMS
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ASSIGNMENT 4

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Problem 1

Let $\tau \in \mathbb{H}$ be a fixed point of an element $\gamma \in \Gamma = \mathrm{SL}_2(\mathbb{Z})$, with $\gamma \neq \pm \mathrm{I}_2$.

1. Show that τ is either equivalent to $\rho = \frac{1}{2} + \frac{\sqrt{-3}}{2}$ or to i (under the action of Γ).
2. Determine the stabilizers Γ_ρ of ρ and Γ_i of i in Γ.
   Note: These points are called *elliptic fixed points*.

Problem 2

Let f be a holomorphic modular form for Γ without any zeroes in $\mathbb{H}$. Then $f(\tau) = c \cdot \Delta(\tau)^m$ for $m \in \mathbb{N}_0$ and $c \in \mathbb{C}$.

Problem 3

1. The derivative of a modular function (a meromorphic modular form of weight 0) is a meromorphic modular form of weight 2.
2. Let $f, g \in M_k$. Then the *Rankin-Cohen-Bracket* $[f, g] := f'g - g'f$ defines a holomorphic modular form of weight $2k+2$. (All derivatives are with respect to $\tau \in \mathbb{H}$.)