McGill University
Department of Mathematics and Statistics
MATH 596: Introduction to modular forms
Winter 2016
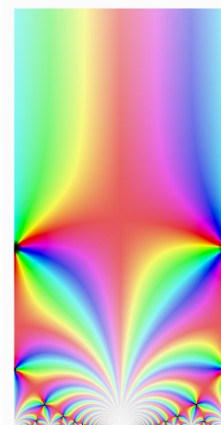
Instructor: Dr. Stephan Ehlen

Location and time: BURN 1234,
Monday and Friday, 1:35pm - 3:25 pm,
Friday 2:30-3:25 is designated as an excercise session,
except for the first week.
(Note: The first class is on Thursday, January 7, 2016,
1:35-3:25.)

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Description: This course is an introduction to the theory of modular forms: holomorphic functions on the complex upper half-plane $\mathbb{H} = \{z \in \mathbb{C} \mid \Im(z) > 0\}$, which are symmetric in a special way with respect to an action of the modular group $\mathrm{SL}_2(\mathbb{Z})$ on $\mathbb{H}$. They appear in various branches of Mathematics and Physics, maybe most prominently (in connection with elliptic curves) in the proof of *Fermat's last theorem*, which states that the equation

$$x^n + y^n = z^n$$

does not have any non-trivial solutions $(x, y, z) \neq (0, 0, 0)$ in the integers for $n \geq 3$.

The course gives an overview of the theory from a classical point of view. During the second half of the semester, we will focus on various methods to construct modular forms: Eisenstein and Poincaré series, theta functions, and *theta lifts* in some special cases.

Syllabus:

1. Elliptic functions
2. Elliptic curves
3. The modular curve as a moduli space
4. Modular forms
5. Poincaré series and Eisenstein series
6. Theta functions
7. Theta lifts and Borcherds products on modular curves

Assessment: Exercise sessions (participation is mandatory).

Assignments: There will be a number of assignments during the semester, which will be discussed in the excercise sessions.

Academic Integrity: McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see http://www.mcgill.ca/integrity/ for more information).

Language Policy: In accord with McGill University's Charter of Students' Rights, students in this course have the right to submit in English or in French any written work that is to be graded.

Extraordinary Circumstances: In the event of extraordinary circumstances beyond the University's control, the content and/or evaluation scheme in this course is subject to change.

References

[DS05] Fred Diamond and Jerry Shurman. *A first course in modular forms*, volume 228 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2005.

[FB09] Eberhard Freitag and Rolf Busam. *Complex analysis*. Universitext. Springer-Verlag, Berlin, second edition, 2009.

[Gun62] R. C. Gunning. *Lectures on modular forms*. Notes by Armand Brumer. Annals of Mathematics Studies, No. 48. Princeton University Press, Princeton, N.J., 1962.

[Ser73] J.-P. Serre. *A course in arithmetic*. Springer-Verlag, New York-Heidelberg, 1973. Translated from the French, Graduate Texts in Mathematics, No. 7.

[Zag08] Don Zagier. Elliptic modular forms and their applications. In *The 1-2-3 of modular forms*, Universitext, pages 1–103. Springer, Berlin, 2008.